

NAG Toolbox for MATLAB

e02ac

1 Purpose

e02ac calculates a minimax polynomial fit to a set of data points.

2 Syntax

```
[a, ref] = e02ac(x, y, m1, 'n', n)
```

3 Description

Given a set of data points (x_i, y_i) , for $i = 1, 2, \dots, n$, e02ac uses the exchange algorithm to compute an m th-order polynomial

$$P(x) = a_1 + a_2x + a_3x^2 + \dots + a_{m+1}x^m$$

such that $\max_i 2|P(x_i) - y_i|$ is a minimum.

The function also returns a number whose absolute value is the final reference deviation (see Section 6). The function is an adaptation of Boothroyd 1967.

4 References

Boothroyd J B 1967 Algorithm 318 *Comm. ACM* **10** 801

Stieffel E 1959 Numerical methods of Tchebycheff approximation *On Numerical Approximation* (ed R E Langer) 217–232 University of Wisconsin Press

5 Parameters

5.1 Compulsory Input Parameters

1: **x(n)** – double array

The values of the x co-ordinates, x_i , for $i = 1, 2, \dots, n$.

Constraint: $x_1 < x_2 < \dots < x_n$.

2: **y(n)** – double array

The values of the y co-ordinates, y_i , for $i = 1, 2, \dots, n$.

3: **m1** – int32 scalar

$m + 1$, where m is the order of the polynomial to be found.

Constraint: **m1** < min(**n**, 100).

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The dimension of the arrays **x**, **y**. (An error is raised if these dimensions are not equal.)
the number n of data points.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **a(m1)** – double array

The coefficients a_i of the final polynomial, for $i = 1, 2, \dots, m + 1$.

2: **ref** – double scalar

The final reference deviation (see Section 6).

6 Error Indicators and Warnings

ifail = 1

m1 $\geq \min(\mathbf{n}, 100)$.

ifail = 2

The constraint $x_1 < x_2 < \dots < x_n$ is violated.

With exact arithmetic the algorithm should terminate after a finite number of steps. This need not necessarily be the case using computer arithmetic. Should the function start cycling then an exit is made with **ref** given a negative value. This is by no means an indicator that a catastrophic error has occurred and does not preclude useful results being obtained.

The absolute value of **ref** is the final reference deviation. See Stieffel 1959 for an explanation of this term.

7 Accuracy

This is wholly dependent on the given data points.

8 Further Comments

The time taken increases with m .

9 Example

```
x = [0;
      0.05;
      0.1;
      0.15;
      0.2;
      0.25;
      0.3;
      0.35;
      0.4;
      0.45;
      0.5;
      0.55;
      0.6;
      0.65;
      0.7;
      0.75;
      0.8;
      0.85;
      0.9;
      0.95;
      1];
y = [1;
      1.051271096376024;
      1.105170918075648;
      1.161834242728283;
      1.22140275816017;
```

```
1.284025416687741;  
1.349858807576003;  
1.419067548593257;  
1.49182469764127;  
1.568312185490169;  
1.648721270700128;  
1.733253017867395;  
1.822118800390509;  
1.915540829013896;  
2.013752707470477;  
2.117000016612675;  
2.225540928492468;  
2.339646851925991;  
2.45960311115695;  
2.585709659315846;  
2.718281828459045];  
m1 = int32(6);  
[a, ref] = e02ac(x, y, m1)  
  
a =  
1.0000  
1.0001  
0.4991  
0.1704  
0.0348  
0.0139  
ref =  
1.0915e-06
```
